Magnetic Viscous Drag for Friction Labs

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The typical friction lab performed in introductory mechanics courses is usually not the favorite of either the student or the instructor. The measurements are not all that easy to make, and reproducibility is usually a troublesome issue. This paper describes the augmentation of such a friction lab with a study of the viscous drag on a magnet sliding down a conducting ramp, e.g., an aluminum ramp (Fig. 1). The measurements are simple and quite reproducible, and it appears to readily catch the interest of students.

While it is apparent to the instructor that slow, constant velocity slide of the magnet down the ramp is produced by magnetic induction, this may well not be apparent to the student.[1] We believe this is *not* a drawback, since our focus is to characterize the behavior of the sliding magnet, *without* delving into the underlying mechanism. Such characterization is a necessary first step in the development of an adequate model of explanation and an important skill for students to develop.

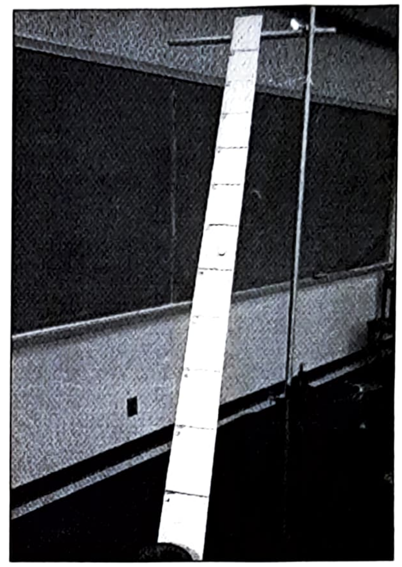

Fig. 1. A magnet slides down an aluminum ramp, having reached terminal velocity within ~2 cm of release point. The angle of incline is varied between 20° and 80°. For each angle the time to travel 1 m is measured and terminal velocity calculated.

Apparatus and procedure

We purchased three aluminum ramps, one 121.92 cm x 7.62 cm x 0.64 cm and two 121.92 cm x 7.62 cm x 0.32 cm for $43 (shipping included). Inexpensive cellophane tape produces a surface with a reasonably constant coefficient of kinetic friction. We were easily able to measure the time for the magnet to slide 1.0 m, for angles ranging between 20° and 80°. While it was visually apparent that the magnet was traveling at a constant velocity, we performed similar timing at shorter distances, for a few angles, and found identical velocities to the ones reported here for slides of 1 m length. We performed three separate runs for each angle, measuring the time for each with two separate stopwatches. For a given angle, the individual timing measurements varied by at most a few percent. We worked with three different aluminum thicknesses: 0.318, 0.635, and 0.953 cm. In order to determine this kinetic coefficient of friction, independent of the drag observed with the aluminum ramp, we also slid the magnet down a similar wooden ramp covered with cellophane tape.

Basic experimental results and drag model

• *Dry Run (no drag)*

There is only one angle of tilt (θ_C) that will cause a neodymium magnet to slide down a wooden ramp at constant velocity, theoretically speaking. For angles greater than this critical angle, the magnet accelerates down the wooden ramp, with an increase in the angle of tilt causing an increase in the acceleration. As the angle of tilt approaches θ_C, the magnet's sliding becomes erratic, with much "stick-and-slip" behavior. In fact, measuring a block's acceleration down an inclined plane is often done as a laboratory exercise to demonstrate how we can characterize the frictional force for dry sliding surfaces.

Such experiments have led us to believe we can (to some degree) model the frictional force as depending only on the character of the surfaces involved and the normal force between them: $F_f = \mu_k F_N$ [Fig. 2(a)]. Applying Newton's second law in this case leads to a constant velocity at only one critical tilt angle (θ_C) given by $\tan(\theta_C) = \mu_k$. At angles greater than θ_C the magnet accelerates with a value given by

$$a = g[\sin(\theta) - \mu_k \cos(\theta)]. \tag{1}$$

We performed this experiment, measuring the time for a 1-cm diameter neodymium magnet to slide ~0.5 m down an inclined wooden ramp. Assuming constant acceleration, and knowing the time and distance of travel, we found the value of the acceleration for two different angles. We obtained two independent measurements of μ_k. These values were within 5% of one another with an average $\mu_k = 0.23$. To ensure that the magnet was sliding across the same surface, whether the ramp was wooden or aluminum, we covered both surfaces with standard transparent tape.

• *Enter viscous drag*

In contrast to the wooden ramp, if you release from rest such a magnet on an inclined aluminum ramp (Fig. 1) it very quickly reaches a relatively small terminal velocity (v_t). The

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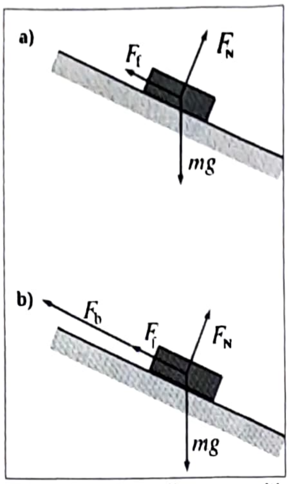

Fig. 2. Free-body diagrams with and without magnetic drag. The top FBD (a) has only dry friction, while the bottom FBD (b) has both dry friction and viscous magnetic drag ($F_B = -bv$).

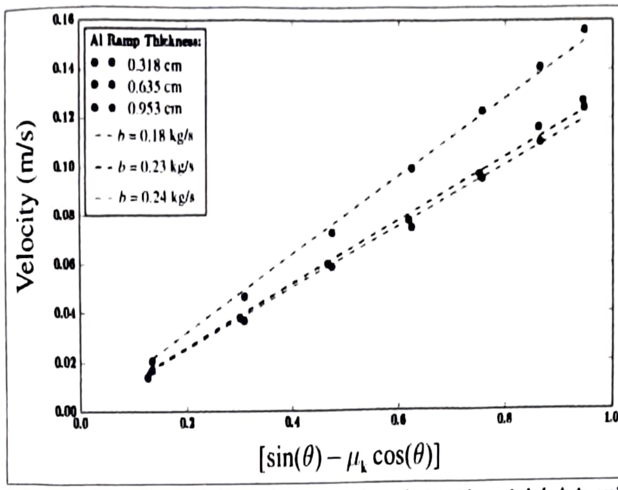

Fig. 3. Comparison of theoretical (- - -) and experimental (···) terminal velocities across a range of ramp angles between 20° and 80°. Theoretical fits were obtained by adjusting the drag parameter *b*. The drag increases as aluminum thickness increases, showing little variation between 0.25 in to 0.375 in.

Results, analysis

In anticipation that the terminal velocity would vary with angle according to Eq. (2), we plotted the terminal velocity versus $[\sin(\theta) - \mu_k \cos(\theta)]$ as shown in Fig. 3 for angles ranging between 20° and 80°. We adjusted the drag parameter *b*, while keeping μ_k at the value obtained from the wooden ramp results to receive the fits shown. The good quality of the fits indicate that the magnet experiences a drag force very nearly proportional to its velocity down the ramp. The increase in the drag parameter with the increase in thickness is indicative of a volume effect, quite distinct from the surface phenomenon of dry sliding friction. Furthermore, the fact that the 0.635-cm and 0.9525-cm thicknesses produce nearly the same drag indicates that the viscous drag is produced in the top ~0.8 cm of the ramp.

lowest v_t was 1.6 cm/s (0.953 cm, 20°), the highest v_t was 15.6 cm/s (0.318 cm, 80°). There is no measurable attraction between the magnet and the aluminum; in fact, we found our tilt angle needed to be less than 80°, since greater angles caused the magnet to "peel off" the aluminum surface. (The magnet's thickness in the presence of the torque provided by the gravitational force causes the magnet to "tip" at angles > 80°.)

The magnet's movement is reminiscent of the slow descent of an object in a viscous fluid: the greater the liquid's viscosity, the lower the descending object's terminal velocity. Situations involving fluid drag are usually left as a secondary topic when friction is discussed in introductory mechanics courses, which is unfortunate. One reason is that fluid drag forces are more important than the dry sliding variety when we need to analyze socially significant situations—fuel efficiency, transport in microorganisms, and climate/weather dynamics. For our purposes we can adopt a simple model in which the drag force is proportional to the velocity: $F_d = -bv$. This model does in fact adequately describe the descent of spherical object "falling" through a viscous fluid at a constant speed, for the appropriate range of object size and viscosity values. Including such a drag force, our revised free-body diagram is shown in Fig. 2(b). Taking the magnet's velocity as its terminal velocity (v_t), we find

$$v_t = \left(\frac{mg}{b}\right)[\sin(\theta) - \mu_k \cos(\theta)]. \tag{2}$$

While it is obvious, for physics teachers, that this drag is produced by induced currents in the aluminum ramp, this line of reasoning is not (necessarily) available to students who are just learning about frictional forces. However, it is *not* necessary that students understand the physical mechanism behind the drag to experimentally explore the phenomenology of the drag.

Physical mechanism

Studies of magnetic drag due to induced eddy currents have focused on the vertical descent of the magnet down a cylindrical tube.[2-4] Our experimental situation lacks the cylindrical symmetry that enables relatively simple quantitative analysis, but we can perform a qualitative study. We limit ourselves to simply showing that the force opposes the magnet's motion, and that the drag force calculated using Faraday's law of induction is proportional to the magnet's velocity.

The magnet's motion during a small time interval is schematically shown in Fig. 4. We take the magnet's field (B_{app}) to be perpendicular to the ramp's surface, with a uniform intensity that abruptly ends at the magnet's edge. Faraday's law states that the emf induced in a closed loop equals the time rate of change of the magnetic field flux. The induced emf causes an induced current (I_{ind}) to flow, where the magnitude of this current depends on the effective resistance (R_{eff}) of the aluminum ramp with respect to this current flow. So we have:

$$\frac{-\Delta\phi}{\Delta t} = emf = I_{ind} R_{eff}. \tag{4}$$

The ramp experiences a force due the magnet's field interacting with the induced currents of nearly free electrons in the aluminum. The simplest expression for this Lorentz force involves a constant current over a length *L* in the presence of a uniform magnetic field:

$$\mathbf{F}_B = I_{\text{ind}}\,\mathbf{L} \times \mathbf{B}. \tag{5}$$

In Fig. 4(a) we see a *top view* of the magnet's position at two closely spaced instants in time ($t_2 > t_1$). The spatial regions that experience a change in flux in this time interval are the "crescent moons" resulting from the magnet leaving the region (on the left) or entering the region (on the right).

In Figs. 4(b)–(f) we see a *side view* of the situation, where we take the magnet's B_{app} to be upward. The profile of the induced magnetic field (B_{ind}), whose direction is gotten from its opposition to the flux change, is shown in Fig. 4(c). The direction of induced current flow (I_{ind}) in the aluminum necessary to produce B_{ind} is shown in Fig. 4(d). [These induced currents would appear as clockwise and counterclockwise loops in the top view shown in Fig. 4(a)]. Figure 4(e) shows time average field of the magnet $\langle B_{\text{app}} \rangle$, averaged between t_1 and t_2. The action of $\langle B_{\text{app}} \rangle$ on I_{ind} produces the Lorentz force shown in Fig. 4(f). The direction of the forces shown can be deduced from application of the cross product indicated in Eq. (5). That $\langle B_{\text{app}} \rangle$ is greater on the inside regions of the "crescents" produces a greater Lorentz force on the currents there, resulting in a net force pointing toward the right in both crescent regions. Since the magnet is shown sliding to the right, we see that it pushes the ramp in the direction of the magnet's travel. Thus (by the third law) the ramp must push the magnet opposite its direction of travel—a drag force.

Is this drag force proportional to the velocity? In our simple model the magnetic field is constant and confined to the space directly below the magnet. Therefore the rate of flux change is due to the rate at which new area is being swept out beneath the traveling magnet. This time rate of area change is simply given by the disk's diameter multiplied by the disk's velocity. Thus we have

$$\frac{\Delta\phi}{\Delta t} = \frac{\Delta(B \cdot A)}{\Delta t} = \frac{B \cdot \Delta A}{\Delta t} = B \cdot D \cdot \frac{\Delta x}{\Delta t} = B \cdot D \cdot v, \tag{6}$$

where D is the diameter of the magnet and v is its velocity. Inserting this result into Eq. (4) shows I_{ind} to be proportional to v, and F_B is proportional to I_{ind} [Eq. (5)], thus F_B is proportional to v. Since F_B acts as a drag force, and since the magnet does travel at a terminal velocity, the model predicts an inductive drag force proportional to the terminal velocity, in good agreement with the experimental results shown in Fig. (3).

Doing more

This simple laboratory situation provides several opportunities for students to perform additional experiments to further characterize the drag on the magnet. The results shown in Fig. 3 clearly show the drag increases with thickness. This behavior could be explored in more detail by experimenting with thinner aluminum stock. Does aluminum foil produce any noticeable drag? The effect on cooling or heating the ramp would also cast light on the mechanism behind the drag force. One could systematically reduce the magnetic effect by using a series of increased tape thicknesses. Two ramps could be used with very small gap between—is the expected decrease in drag force observed? One could also increase precision by using a wider tape with a well-known coefficient of friction, such as Kapton. Students could design ways to measure the time using something more sophisticated than a stopwatch. What occurs when you stack magnets?

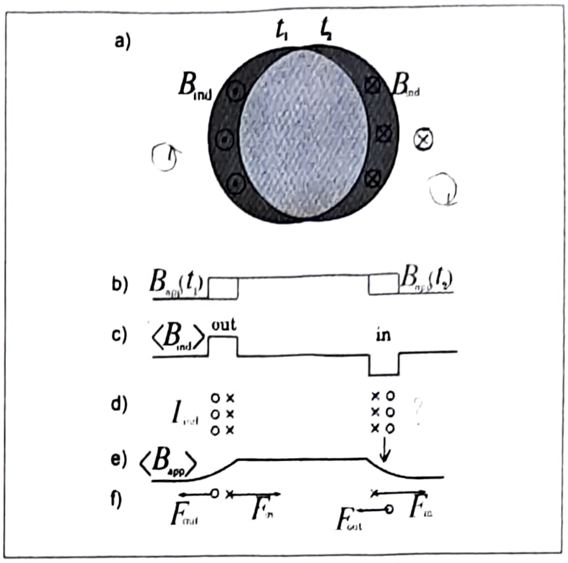

Fig. 4. Cross sections of magnetic fields, induced currents, and magnetic forces in the aluminum ramp. (a) Seen from above, the magnetic field produced by the magnet is shown at two nearby instants in time ($t_2 > t_1$) as the magnet slides to the right. The field (B_{app}) points out of the page. Applying Lenz's law results in the induced fields shown (B_{ind}). (b) Side view of magnet's field at the two instants. (c) The induced magnetic field, averaged over the time interval. (d) The rotation direction of induced current in the ramp, necessary to produce B_{ind}. (e) The magnet's field averaged over the time interval. (f) The direction and relative magnitudes of the magnet field forces on the induced currents in the ramp.

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